Physics 208 Final Exam

Name (printed)_____*Solution*_____

On my honor as a Texas A&M University at Qatar student, I will neither give nor receive unauthorized help on this exam.

Name (signed) _____*Solution!*_____

You are graded on your work, with partial credit. (Be sure to include the proper units in each answer.)
The answer by itself is not enough, and you receive credit only for your work.
Please be clear and well-organized, so that we can easily follow each step of your work.
See the last page of the exam for the formula sheets.

1. (20) A thin slit illuminated by light of wavelength λ produces its first dark band at $25°$ in air. When the entire apparatus (slit, screen, and space in between) is immersed in an unknown transparent liquid, the slit's first dark band occurs instead at $18°$. Calculate the refractive index of the liquid.

dark band: $a \sin\theta = m\lambda = \lambda$ for $m = 1$ in liquid

In air, $a \sin\theta_0 = m\lambda_0 = \lambda_0$ for $m = 1$ again

Also, $\lambda = \dfrac{\lambda_0}{n}$ [since $v = \lambda f$ & $c = \lambda_0 f \Rightarrow \dfrac{\lambda}{\lambda_0} = \dfrac{v}{c} = \dfrac{1}{n}$]

So $\dfrac{a \sin\theta}{a \sin\theta_0} = \dfrac{\lambda}{\lambda_0} = \dfrac{1}{n}$

and $\boxed{n =}\dfrac{\sin\theta_0}{\sin\theta} = \dfrac{\sin 25°}{\sin 18°} = \dfrac{0.423}{0.309} = \boxed{1.37}$

[other version: $\dfrac{\sin 35°}{\sin 19°} = \dfrac{0.574}{0.326} = \boxed{1.76}$]

2. An electromagnetic wave has an electric field given by
$$\bar{E}(z,t) = \left(6.0 \times 10^4 \text{V/m}\right) \hat{j} \, \cos\left(kz - \left(4.0 \times 10^{12}\,\text{rad/s}\right)t\right).$$

(a) (4) In which direction is the wave traveling? Explain.

positive z direction,
since z increases as t increases
for fixed value of $\bar{E}(z,t)$
[other version : positive y direction]

(b) (4) Calculate the frequency f and period of oscillation T at a given point.

Answers: $f = \underline{6.37 \times 10^{11}\ Hz}$ $T = \underline{1.57 \times 10^{-12}\ s}$

one way: $\omega = 2\pi f = 4.0 \times 10^{12}\ rad/s$

$\Rightarrow \boxed{f = \dfrac{\omega}{2\pi}} = \dfrac{4.0 \times 10^{12}\ rad/s}{2\pi} = \boxed{6.37 \times 10^{11}\ Hz}$

$= 637\ GHz$

and $\boxed{T = \dfrac{1}{f}} = \boxed{1.57 \times 10^{-12}\ s} = \boxed{1.57\ ps}$

[other version : $f = \boxed{1.27 \times 10^{12}\ Hz} = 1.27\ THz,$

$T = \boxed{7.85 \times 10^{-13}\ s} = 0.785\ ps$]

(c) (4) Calculate the wavelength of this electromagnetic wave.

Answer: wavelength = $\underline{4.71 \times 10^{-4}\ m}$

$c = \lambda f \Rightarrow \boxed{\lambda = \dfrac{c}{f}} = \dfrac{3.00 \times 10^8\ m/s}{6.37 \times 10^{11}\ Hz} = \boxed{4.71 \times 10^{-4}\ m}$

$= 0.471\ mm$

[other version : $\lambda = \boxed{2.36 \times 10^{-4}\ m}$]

(d) (4) Calculate the magnitude of the maximum value of the magnetic field.

Answer: magnitude of maximum value of magnetic field = $2.0 \times 10^{-4}\ T$

$E = cB \Rightarrow \boxed{B = \dfrac{E}{c}} = \dfrac{6.0 \times 10^4\ V/m}{3.00 \times 10^8\ m/s} = \boxed{2.0 \times 10^{-4}\ T}$

[other version: $\boxed{1.3 \times 10^{-4}\ T}$]

Check on units: $F = qE$ and $F = qvB$

$\Rightarrow \cancel{C} \cdot \dfrac{V}{m} = \cancel{C} \cdot \dfrac{m}{s} \cdot T \Rightarrow T = \dfrac{V/m}{m/s}$

(e) (4) Write the vector equation for the magnetic field $\bar{B}(z,t)$.

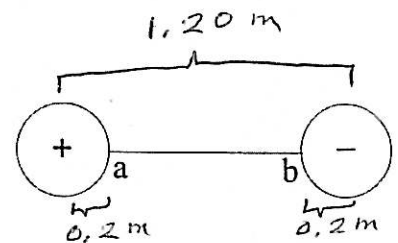Answer: $\bar{B}(z,t) = \underline{-\left(2.0 \times 10^{-4}\ T\right) \hat{i}\ \cos\left(kz - (4.0 \times 10^{12}\ rad/s)t\right)}$

using right-hand rule to get direction
(i.e., minus sign)

[other version : $\boxed{+\left(1.3 \times 10^{-4}\ T\right) \hat{i}\ \cos\left(ky - (8.0 \times 10^{12}\ rad/s)t\right)}$]

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3. Two oppositely charged, identical insulating spheres, each 40 cm in diameter and carrying a uniform charge of magnitude 400 μC, are placed 1.20 m apart, center to center.

1.20 m

+ a   b −

0.2 m   0.2 m

(a) (18) If a voltmeter is connected between the nearest points (a and b) on their surfaces, what will it read? (As always, show the steps in your calculation.)

$V = $ sum of $k\frac{q}{r}$ for both spheres , $k = \frac{1}{4\pi\epsilon_0}$

$V_a = (9.0 \times 10^9)\left(\frac{400\times10^{-6}}{0.2} - \frac{400\times10^{-6}}{1.0}\right)$ with units omitted in intermediate steps,

$= 3.6 \times 10^6 (5-1)$ 20 cm = 0.2 m,

$= 14.4 \times 10^6$ V $1\mu C = 10^{-6} C$

and $V_b = -14.4 \times 10^6$ V

then   potential difference $= \boxed{28.8 \times 10^6 \text{ V}}$

[other version:

$V_a = (9.0 \times 10^9)\left(\frac{200\times10^{-6}}{0.1} - \frac{200\times10^{-6}}{0.7}\right)$

$= (1.8 \times 10^6)(10 - 1.43)$

$= 15.4 \times 10^6$ V

$V_b = -15.4 \times 10^6$ V

and   pot. diff. $= \boxed{30.8 \times 10^6 \text{ V}}$]

(b) (2) Which point, a or b , is at the higher potential? Explain how you can know this without any calculation.

a is at higher potential because it is closer to positive charge

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4. A deuteron (the nucleus of ^{2}H, the isotope of hydrogen called deuterium) has a mass of 3.34×10^{-27} kg and a charge of 1.60×10^{-19} C. The deuteron travels in a circular path with a radius of 12.0 mm, in a magnetic field with a magnitude of 2.0 T.

(a) (10) Set the centripetal force mv^2 / R equal to the magnetic force, and obtain the expression for the speed in terms of the radius of the orbit, the magnetic field, the mass of the deuteron, and its charge.

$$\frac{m v^2}{R} = q v B \Rightarrow \boxed{v = \frac{q B R}{m}}$$

(b) (5) Use your result from part (a) to calculate the speed of the deuteron.

Answer: speed = $\underline{1.15 \times 10^6 \ \frac{m}{s}}$

$$\boxed{v =} \ \frac{(1.60 \times 10^{-19} C)(2.0 T)(12 \times 10^{-3} m)}{3.34 \times 10^{-27} kg}$$

$$= \boxed{1.15 \times 10^6 \ \frac{m}{s}}$$

$$[other \ version: \ 6.47 \times 10^5 \ \frac{m}{s}]$$

(c) (5) Calculate the potential difference (in volts) through which the deuteron would have to be accelerated to acquire this speed.

Answer: potential difference = $\underline{1.38 \times 10^4 \ V}$

$$q \Delta V = \frac{1}{2} m v^2 = \frac{1}{2}(3.34 \times 10^{-27} kg)(1.15 \times 10^6 \ \frac{m}{s})^2$$

$$= 2.21 \times 10^{-15} \ J$$

$$\Rightarrow \boxed{\Delta V =} \ \frac{2.21 \times 10^{-15} \ J}{1.60 \times 10^{-19} \ C} = \boxed{1.38 \times 10^4 \ V}$$

$$[other \ version: \ 4.37 \times 10^3 \ V]$$

5. In an LC circuit, the inductance is 90 mH and the capacitance is 3.0 μF. During the oscillations of the circuit, the maximum current in the inductor is 0.80 mA.

(a) (10) Calculate the maximum charge on the capacitor.

Answer: maximum charge = $\underline{0.416\ \mu C\quad or\quad 4.16 \times 10^{-7} C}$

$$\frac{1}{2}Li^2 + \frac{1}{2C}q^2 = \frac{1}{2}Li_{max}^2 = \frac{1}{2C}q_{max}^2$$

$$\Rightarrow \boxed{q_{max} = \sqrt{LC}\ i_{max}}\ \underset{\text{arithmetic given below in part (b)}}{\longleftarrow}$$

$$= (5.2 \times 10^{-4}\ s)(0.80 \times 10^{-3}\ A)$$

$$= \boxed{4.16 \times 10^{-7}\ C = 0.416\ \mu C}$$

[other version: $q_{max} = (5.29 \times 10^{-4})(0.60 \times 10^{-3})\ C = \boxed{0.317\ \mu C}$]

(b) (10) Calculate the magnitude of the charge on the capacitor at the instant (time t) when the current in the inductor has magnitude 0.40 mA.

Answer: magnitude of charge at this instant = $\underline{0.36\ \mu C\quad or\quad 3.6 \times 10^{-7} C}$

$$\frac{1}{2}Li^2 + \frac{1}{2C}q^2 = \frac{1}{2}Li_{max}^2$$

$$\Rightarrow \frac{1}{2C}q^2 = \frac{1}{2}L(i_{max}^2 - i^2)$$

$$\Rightarrow q^2 = LC(i_{max}^2 - i^2)$$

$$\Rightarrow \boxed{q = \sqrt{LC}\ \sqrt{i_{max}^2 - i^2}}\qquad [\text{recalling that } \sqrt{LC} \text{ has units of } s]$$

$$= \sqrt{(90 \times 10^{-3}\ H)(3.0 \times 10^{-6}\ F)}$$

$$\times \sqrt{(0.80 \times 10^{-3}\ A)^2 - (0.40 \times 10^{-3}\ A)^2}$$

$$= (5.2 \times 10^{-4}\ s)\sqrt{0.64 - 0.16} \times 10^{-3}\ A$$

$$= \boxed{3.6 \times 10^{-7}\ C}$$

$$= \boxed{0.36\ \mu C}$$

Of course, it is simpler to note that

$$\frac{q}{q_{max}} = \sqrt{\frac{i_{max}^2 - i^2}{i_{max}^2}} = \sqrt{1 - \left(\frac{i}{i_{max}}\right)^2} = \sqrt{1 - \frac{1}{4}} = 0.866$$

$$\Rightarrow q = 0.866\ q_{max} = 0.36\ \mu C$$

[other version: $q = \boxed{0.274\ \mu C}$]

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6. Two radio antennas are separated by a distance of $d = 8$ m. They broadcast at a frequency of 80 MHz (with 1 MHz $= 10^6$ Hz). Let θ be the angle shown in the figure, between a perpendicular line, from the midpoint between the antennas, and a line to the observation point P, again from this midpoint. The drawing is not to scale, and you may assume that $R \gg d$, so that this is equivalent to the usual 2-slit experiment.
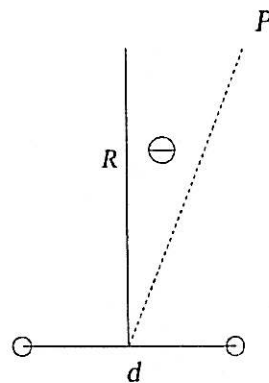
Let us remind you of the general result for the intensity I as a function of the phase difference ϕ between the waves received from two sources, with ϕ related to the path length difference Δr in the usual way: $\dfrac{\phi}{2\pi} = \dfrac{\Delta r}{\lambda}$, where λ is the wavelength:

$$I = I_0 \cos^2\left(\frac{\phi}{2}\right).$$

(a) (4) Calculate the wavelength of the waves emitted by these antennas.

$$\boxed{\lambda =}\frac{c}{f} = \frac{3.00 \times 10^8 \, m/s}{80 \times 10^6 \, Hz} = \boxed{3.75 \, m}$$

$[$other version$: \boxed{6.00 \, m}]$

(b) (4) Recalling that this is like a 2-slit experiment, calculate the path length difference for $\theta = 5°$.

$$\boxed{\Delta r =} d \sin \theta$$
$$= (8 \, m) \sin 5°$$
$$= \boxed{0.697 \, m}$$

$[$other version $: \boxed{0.628 \, m}]$

(c) (4) For $R = 800$ m, the intensity along the perpendicular line, with $\theta = 0$, is $0.12 \, W/m^2$. Calculate the intensity at this same distance for $\theta = 5°$.

$(rad) \longrightarrow \dfrac{\phi}{2\pi} = \dfrac{\Delta r}{\lambda} \Rightarrow \boxed{\phi =} (2\pi \, rad) \dfrac{0.697 \, m}{3.75 \, m} = \boxed{1.17 \, rad}$

$$\text{Then } \boxed{I =} (0.12 \, \tfrac{W}{m^2}) \cos^2\left(\frac{1.17 \, rad}{2}\right)$$
$$= (0.12 \, \tfrac{W}{m^2})(0.834)^2$$
$$= \boxed{0.0834 \, \tfrac{W}{m^2}}$$

$[$other version$: \phi = 0.658 \, rad$
$$I = (0.04 \, \tfrac{W}{m^2})(0.946)^2$$
$$= \boxed{0.0358 \, \tfrac{W}{m^2}}]$$

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(d) (4) At this same distance (800 m), calculate the smallest value of θ for which $I = \frac{1}{2}I_0$.

$45°$, but ϕ in rad

$$\frac{1}{2}I_0 = I_0 \cos^2\left(\frac{\phi}{2}\right) \Rightarrow \cos\left(\frac{\phi}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\phi}{2} = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4} \text{ rad}$$

$$\Rightarrow \phi = \frac{\pi}{2} \text{ rad}$$

Then $\frac{d\sin\theta}{\lambda} = \frac{\phi}{2\pi} = \frac{\pi/2}{2\pi} = \frac{1}{4} \Rightarrow \boxed{\sin\theta = \frac{1}{4}\frac{\lambda}{d}}$

$\sin\theta = \frac{1}{4}\frac{3.75m}{8m} = 0.117 \Rightarrow \boxed{\theta = 6.7° \text{ or } 0.117 \text{ rad}}$

[other version: $\sin\theta = \frac{1}{4}\frac{6.00 m}{12 m} = 0 \Rightarrow \boxed{\theta = 7.2°} \text{ or } 0.125 \text{ rad}$]

(e) (4) Calculate all the values of θ for which $I = 0$. (You may continue to use the usual small-angle approximation that we used for the 2-slit experiment, even though there will be some inaccuracy at the largest angles.)

$$0 = \cos\left(\frac{\phi}{2}\right) \quad \text{and} \quad \frac{\phi}{2\pi} = \frac{\Delta r}{\lambda} = \frac{d\sin\theta}{\lambda} \Rightarrow \boxed{\sin\theta = \frac{\lambda}{d}\frac{\phi}{2\pi}}$$

① $\frac{\phi}{2} = \frac{\pi}{2}$ ← [$90°$, but ϕ in radians] $\Rightarrow \phi = \pi$

$$\Rightarrow \sin\theta = \frac{\lambda}{d}\frac{\pi}{2\pi} = \frac{1}{2}\frac{\lambda}{d} = \frac{1}{2}\frac{3.75 m}{8m} = 0.234$$

$$\Rightarrow \boxed{\theta = 0.236 \text{ rad} \quad \text{or} \quad 13.5°}$$

② $\frac{\phi}{2} \approx \frac{\pi}{2} + \pi = \frac{3\pi}{2}$

$$\Rightarrow \sin\theta = (3)(0.234) = 0.702$$

$$\Rightarrow \boxed{\theta = 44.6°}$$

③ $\frac{\phi}{2} = \frac{\pi}{2} + 2\pi = \frac{5\pi}{2}$

$$\Rightarrow \sin\theta = (5)(0.234) = 1.17$$

which is impossible

[other version: ① $\sin\theta = \frac{1}{2}\frac{\lambda}{d} = \frac{1}{2}\frac{6.00 m}{12 m} = 0.25$

$\Rightarrow \theta = \boxed{14.5°}$

② $\sin\theta = 0.75$

$\Rightarrow \theta = \boxed{48.6°}$

③ $\sin\theta = 1.25$, which is impossible]

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7. An insulating hollow sphere has inner radius r_1 and outer radius r_2. Within the insulating material, the volume charge density is given by $\rho(r) = \dfrac{c}{r}$, where c is a positive constant.

Please give your answers below in terms of r_1, r_2, c, and ε_0.

(a) (1) Draw a picture of this hollow sphere, showing r_1 and r_2.
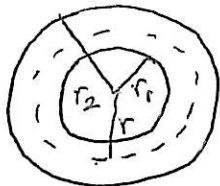
(b) (6) In terms of the above constants, calculate the charge $q(r)$ contained within a spherical surface of radius r, where $r_1 < r < r_2$.

$$\boxed{q(r) =} \int_{r_1}^{r} \rho(r') \cdot 4\pi r'^2 \, dr'$$

$$= \int_{r_1}^{r} \frac{c}{r'} \cdot 4\pi r'^2 \, dr'$$

$$= 4\pi c \int_{r_1}^{r} r' \, dr'$$

$$= 4\pi c \left[\frac{r'^2}{2} \right]_{r_1}^{r}$$

$$= \boxed{4\pi c \cdot \frac{1}{2} \left(r^2 - r_1^2 \right)}$$

(c) (6) Again in terms of the above constants, calculate the magnitude $E(r)$ of the electric field at a distance r from the center of the shell, where again $r_1 < r < r_2$.

Gauss's law: $\oint \vec{E} \cdot d\vec{A} = q(r)/\varepsilon_0$

$$\Rightarrow 4\pi r^2 \cdot E(r) = \frac{4\pi c}{\varepsilon_0} \cdot \frac{1}{2} \left(r^2 - r_1^2 \right)$$

$$\Rightarrow \boxed{E(r) = \frac{c}{2\varepsilon_0} \left(1 - \frac{r_1^2}{r^2} \right)}$$

(d) (6) A point charge Q is placed at the center of the hollow space, at $r = 0$. In terms of the above constants, calculate the required value of Q (sign and magnitude) in order for the electric field to be constant in the region $r_1 < r < r_2$.

$$E_Q = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

Want
$$E_Q + E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} + \frac{c}{2\epsilon_0}\left(1 - \frac{r_1^2}{r^2}\right) = \text{constant}$$

and this requires

$$\frac{Q}{4\pi\epsilon_0} \frac{1}{r^2} - \frac{c r_1^2}{2\epsilon_0} \frac{1}{r^2} = 0$$

or
$$\frac{Q}{4\pi\epsilon_0} - \frac{c r_1^2}{2\epsilon_0} = 0$$

or
$$\boxed{Q = 2\pi c r_1^2}$$

(e) (1) What is the value of the constant field in this region?

Then
$$\boxed{E_Q + E = \frac{c}{2\epsilon_0}}$$.

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8. (5 points extra credit) Describe how you could achieve each of the following, using the ideas in our course. **Be precise in specifying the relevant parameters – e.g., thickness of coating or orientation for lenses of sunglasses.**

(i) A thin coating on an automobile window, so that as much as possible of the light from outside is transmitted inside rather than reflected, and you can perhaps see more clearly.

We get destructive interference
for reflected light at normal incidence
if t is chosen properly.

air
film $\Big\}t$
glass
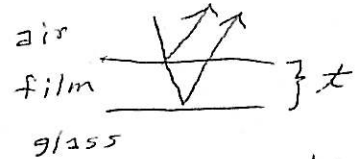
For the case $n_{film} < n_{glass}$, there are two phase reversals
and dest. int. for $2t = \frac{1}{2}\lambda$ or $t = \frac{1}{4}\lambda$,
where $\lambda = \frac{\lambda_0}{n_{film}}$.

(ii) Sunglasses that will block much of the glare of sunlight off water.

polarizing lenses, with light absorbed if
the electric field is parallel to
the water surface

9. (5 points extra credit) Estimate the size of the pupil of one of your eyes. That is, estimate the diameter of the opening in one of your eyes that permits light to enter the eye and fall on the retina. Then estimate the best possible angular resolution that your eye can achieve, using the Rayleigh criterion

$$\sin\theta = 1.22\frac{\lambda}{D}.$$

Since this is an extra credit problem, you will have to remember roughly what is the wavelength of visible light. Every really good engineer should have numbers like this in his or her head.

If $D \sim 1\,mm = 10^{-3}\,m$ and $\lambda \sim 500\,nm = 500\times10^{-9}\,m$

$\sin\theta \sim \dfrac{500\times10^{-9}\,m}{10^{-3}\,m} = 5\times10^{-4}$

$\Rightarrow \boxed{\theta \sim 0.03^\circ}$

Have a good summer!

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